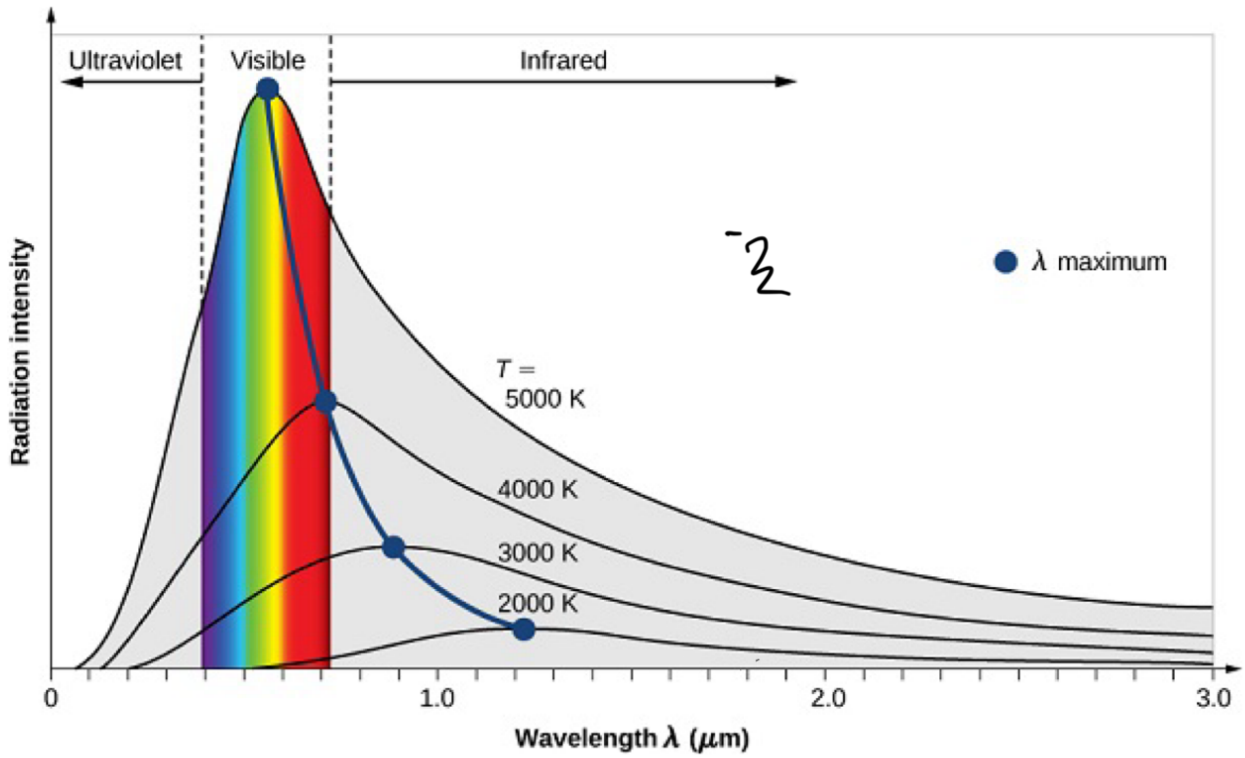

Spectral distribution of Black body radiations

from Lummer & Pringsheim (1899) experiment

Results: (1) At a particular temp., the distribution of energy is not uniform among various wave lengths of the radiation emitted by the black body.

(2) For each temperature, there is a wave length (λ_m) at which the energy radiated is max. (E_m)

(3) With increase of temp., the maxima shifts higher but towards lower wave length i.e. with ↑temp corresponding λ_m decreases

(4) Higher the temperature more pronounced is

the maxima.

(5) The area under the curve for a particular temperature gives the total energy emitted by the black body per unit area per second for the complete spectrum — i.e. corresponding to all the wave lengths at that particular temperature (from $\lambda = 0$ to $\lambda = \infty$).

(6) The area under the curve increases with increase of temperature & it is found that the area is proportional to the fourth power of the absolute temperature.

As the area under the curve represents the total energy emitted by the black body per unit area per second, hence

$$E_B \propto T^4$$ ——— Stefan Boltzmann's Law

(7) From the curves it is found ———

$$\lambda_{m_1} T_1 = \lambda_{m_2} T_2 = \lambda_{m_3} T_3$$

$$\Rightarrow \lambda_m T = \text{Constant}$$

$$\boxed{\lambda_m \propto \frac{1}{T}}$$ → Wien's displacement Law

The wave length λ_m for which the black body is maximum is

emittance of a black body (.
inversely proportional to its Absolute Temp. — Wein's displacement law

Why color of visible region radiation changes from red (high λ) → yellow ——→ as temp is increased.

Explanation

Spectral Distribution on Classical Mechanics —

Using Maxwell's electromagnetic theory

the source of radiant energy (→ electromagnetic waves) are the vibrating particles like oscillators & these oscillators could radiate or absorb any amount of energy. The frequency (ν) of the energy / wave emitted is equal to the frequency of the oscillator

(or ν)

Based on these Classical concepts

Wein (1896)

Rayleigh & Jeans (1900)] failed to explain the distribution of b.b.r. using classical concepts of continuous emission of radiation.

But Wein's eqn → wasable to explain At Low wave lengths only

$$E_\lambda = \frac{8\pi hc}{\lambda^5} e^{-hc/KT\lambda}$$

At high wave lengths

Rayleigh & Jeans → [illegible]

$$E_\lambda = \frac{8\pi}{\lambda^4} kT$$

E_λ → emissive power of b.b. at wavelength λ, c → vel. of light
h → Planck's const., K → Boltzmann const = R/N, T → Ab. Temp.

Neither of the two was able to explain the complete distribution of b.b.r.

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(4)

Planck's Radiation Law. (Max Planck 1900)

Oscillator of a b.b can't have any amount of energy, but only have discrete amount of energy depending upon the freq. of the oscillator.

"Energy is emitted or absorbed not continuously but discontinuously in the form of packets of energy called Quanta. The Energy of each quantum is given by the relation $E = h\nu$, where ν → freq of radiation & h → Planck's Constant.

$E = h\nu$

Thus total energy emitted or absorbed is either unit quantum i.e. $1h\nu$ or a whole number multiple of $h\nu$. $(nh\nu)$.

Based upon above concepts, Planck deduced expression for the energy (E_λ), radiated by a black body at wave length λ —

$$E_\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/KT\lambda} - 1}$$

Planck's Radiation Law

① if λT is small, $e^{hc/KT\lambda} \gg 1$, so that denominator in above eqn can be neglected.

$$E_\lambda = \frac{8\pi hc}{\lambda^5} e^{-hc/KT\lambda}$$

Wien's law

② If λT is large, $e^{-hc/KT\lambda}$ may be expanded by the exponential theorem

i.e. $e^{hc/KT\lambda} = 1 + \frac{hc}{KT\lambda} + 2! \ldots + 3 - \cdots$

So that Planck's radiation law equation reduces to

$$E_\lambda = \frac{8\pi hc}{\lambda^5} \times \frac{KT\lambda}{hc}$$

$$E_\lambda = \frac{8\pi}{\lambda^4} \cdot KT$$

→ Rayleigh-Jeans Law

Derivation of Planck's Radiation law —

Planck considered bbr (hohlraum) to be made up of linear oscillators of molecular dimensions & Energy of a linear oscillator have discrete values — 0, $1h\nu$, $2h\nu$, $3h\nu$ ..... $nh\nu$.

If N_0, N_1, N_2, N_3 ..... N_n are the number of oscillators per unit volume of the hohlraum with Energies — 0, $1h\nu$, $2h\nu$ $3h\nu$ ... respectively.

Then Total number of oscillators per unit volume will be —

$$N = N_0 + N_1 + N_2 + N_3 + \cdots \quad (A)$$

But the number of oscillators N_r, having energy E_r is given by (Maxwell's formula)

$$N_r = N_0 \cdot e^{-E_r/KT}$$

$$N_1 = N_0 e^{-h\nu/KT}$$

$$N_2 = N_0 e^{-2h\nu/KT}$$

Putting these values in equation (A)

$$N = N_0 + N_0 e^{-h\nu/KT} + N_0 e^{-2h\nu/KT} + \cdots N_0 e^{-nh\nu/KT}$$

$$= N_0 \left(1 + e^{-h\nu/KT} + e^{-2h\nu/KT} + \cdots\right)$$

$$= \frac{N_0}{(1-e^{-h\nu/kT})} \qquad (1-e^{-x})^{-1} = 1 + e^{-x} + e^{-2x} + \cdots$$

The total energy of N oscillators will be $-h\nu/kT = x$

$$E = N_1 h\nu + N_2 \cdot 2h\nu + N_3 \cdot 3h\nu + \cdots$$

$$= h\nu \left[N_1 + 2N_2 + 3N_3 + \cdots \right]$$

$$= h\nu \left[N_0 \cdot e^{-h\nu/kT} + 2N_0 e^{-2h\nu/kT} + 3N_0 e^{-3h\nu/kT} + \cdots \right]$$

$$= N_0 h\nu \left[e^{-h\nu/kT} + 2e^{-2h\nu/kT} + 3e^{-3h\nu/kT} + \cdots \right]$$

$$= N_0 h\nu e^{-h\nu/kT} \left[1 + 2e^{-h\nu/kT} + 3e^{-2h\nu/kT} + \cdots \right] \qquad \text{If } x = e^{-h\nu/kT}$$

$$= N_0 h\nu e^{-h\nu/kT} \left[1 + 2x + 3x^2 + 4x^3 + \cdots \right]$$

$$= N_0 h\nu e^{-h\nu/kT} \left[1 - x \right]^{-2}$$

$$(1-x)^{-2} \rightarrow 1 + 2x + 3x^2 + \cdots$$

$$= \frac{N_0 h\nu e^{-h\nu/kT}}{(1-x)^2}$$

$$E = \frac{N_0 h\nu e^{-h\nu/kT}}{\left(1 - e^{-h\nu/kT}\right)^2}$$

x ... per oscillator is

Average energy per

$$\bar{E} = \frac{E}{N} = N_0 h\nu \frac{e^{-h\nu/kT}}{(1-e^{-h\nu/kT})^2} \times \frac{1-e^{-\frac{h\nu}{kT}}}{N_0}$$

$$E = \frac{h\nu\, e^{-h\nu/kT}}{1-e^{-h\nu/kT}} = \boxed{\frac{h\nu}{e^{h\nu/kT}-1}} = \bar{E} \quad (B)$$

On dividing numerator & denominator by $e^{-h\nu/kT}$

⇒ Average energy of the oscillator is not kT (Classical Concept) but equal to $\left(h\nu / e^{h\nu/kT}-1\right)$ → According to Planck's quantum theory

no. of oscillators / volume having freq. in the range $\nu \to \nu + d\nu$

Learn $\boxed{\frac{8\pi\nu^2 d\nu}{c^3}}$ — (C)

The average energy per unit volume (energy density) inside the enclosure is obtained by multiplying

eqn B with ν

$$E_\nu d\nu = \frac{8\pi \nu^2 d\nu}{c^3} \times \frac{h\nu}{e^{h\nu/kT} - 1}$$

Putting $\nu = \frac{c}{\lambda}$ & $d\nu = -\frac{c}{\lambda^2} d\lambda$,

The average energy per unit volume in the enclosure for the wave length between λ & $\lambda + d\lambda$

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{h\nu/kT} - 1} d\lambda$$

Energy radiated by the black body corresponding to wave length λ ——

$$E_\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/kT\lambda} - 1}$$

Planck's Radeation Law.

$C_1 = 8\pi hc$, $C_2 = hc/k$

$$E_\lambda = C_1 \lambda^{-5} \frac{1}{e^{C_2/\lambda T} - 1}$$

Experimental Verification

Iso Thermal method

Same Principle — Lummer & Brodhun

$\lambda_m \rightarrow$ max emission

C_1 & C_2

21.10.2025

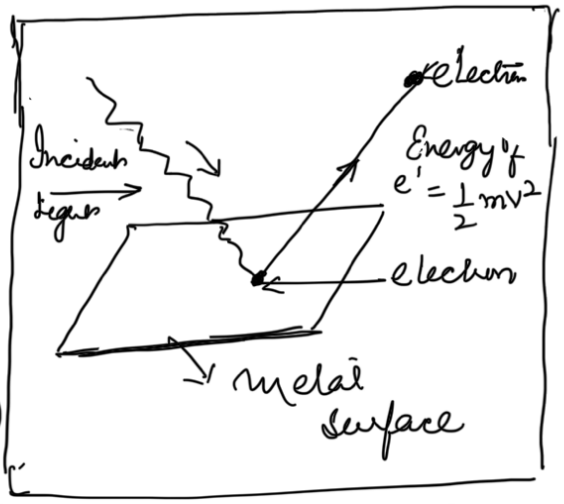
31.10.

Photoelectric Effect :-

When a beam of light with frequency equal to or greater than a particular value (threshold freq.) is allowed to strike the surface of a metal, electrons are ejected instantaneously from the surface of the metal. This effect is called "Photoelectric effect".

Main observations:

* The electrons are ejected only if the frequency of the incident light is equal to or greater than a minimum value, called threshold freq. (ν_0)



Photoelectric Effect

* The e' are ejected instantaneously i.e. there is no time interval between hitting the metal surface by the light & emission of e's.

* The K.E of emitted e's $\propto$ freq. of the incident light.

* The no of e' emitted (ejected) $\propto$ to the intensity of the incident light.

Accord'g to classical theory, the energy of light depends upon its intensity. $\Rightarrow$ if the metal surface is exposed to light continuously, to the light of any frequency, the e's should

Keep on gaining energy & ultimately the energy of should become so high that the e' should leave the metal atom. But this doesn't happen.

However, Einstein explained different observations about photoelectric effect by applying Planck's Quantum theory.

Each quantum of light called Photon has energy equal to $h\nu$.

When photon hits metal atom, it transfers its energy to e'. Energy equal to threshold value is used up for bringing about ionisation (release of e') & Remainings energy is converted into the K.E. of the e'. (the quantity $h\nu_0$ → work function equal to ionization of metal atom $h\nu_0 = IE$)

$$\Rightarrow h\nu = h\nu_0 + \frac{1}{2}mv^2$$

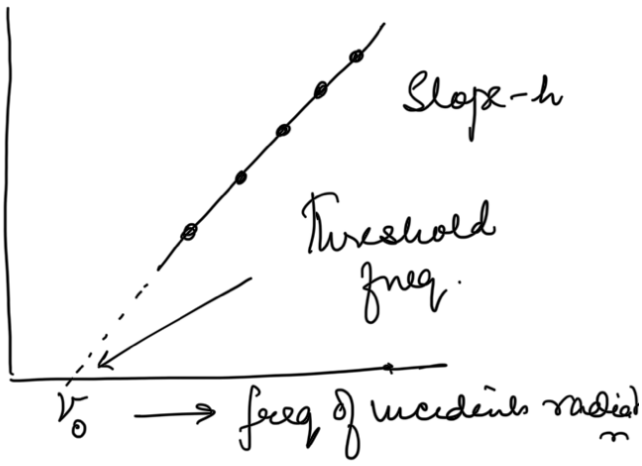
So, if the frequency of incident light is equal to the threshold value, e' will be emitted without any Kinetic energy

Also, intensity of light → no. of photons hitting the metal surface per unit time. Thus increase in intensity increases the no. of e's emitted but will have no effect on their Kinetic energy.

eqn can be written as —

$$KE = h\nu - h\nu_0 = \frac{1}{2}mv^2$$

This eqn shows that K.E of the e's emitted varies linearly with freq of the incident radiation.



⇒ Plot of K.E of emitted e' Vs freq. of incident radiation will be a straight line with slope equal to Planck's constant, h.

This is one of the methods to calculate the value of Planck's constant.

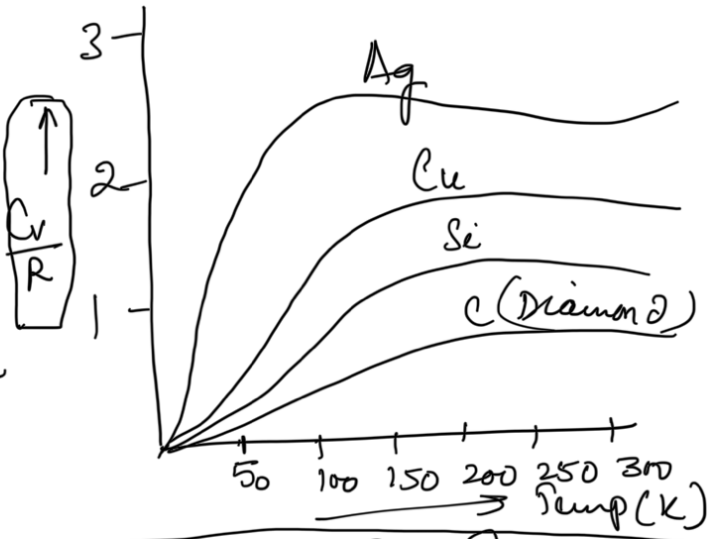
Heat Capacity of solids

Using Classical Physics, it is found that the heat capacity of all monochromatic solids (metals) should be constant & equal to 3R. (Dulong & Petit's law).

However, experimentally it is found to be true only at high temperature.

It is found to be less than 3R &

At low temp. the value is found to be less than 3R
the values approach zero at $T \to 0$.



Variation of molar heat Capacity with Temp for Ag, Cu, Si & C.

Explanation:

A monoatomic solid can be considered as a collection of oscillators each having three degrees of freedom.

According to classical Law of Equipartition of Energy, each atom (oscillator) in the solid has mean vib energy = 3kT.

⇒ 1 mole of monochromatic solid (having Avogadro's no of atoms, N_A) will have total vib. Energy i.e. molar internal energy equal to

$$E = N_A (3kT) = 3RT \quad (\because N_A k = R).$$

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V = 3R$$

Using concepts of Planck's quantum theory, Einstein suggested that all the oscillators don't vibrate with the same freq. ν i.e. they don't have same vibrational energy ($h\nu$) but may

have energies which are integral multiples of some minimum value, $h\nu_0$ $(= n h\nu_0)$. The mean energies of the oscillators is given as —

$$\bar{E} = \frac{h\nu_0}{e^{h\nu_0/KT} - 1}$$

Molar energy of the solid will be

$$E = N_A \times 3\bar{E} = \frac{3N_A h\nu_0}{e^{h\nu_0/KT} - 1} \quad \left(\text{instead of } 3RT\right)$$

$$\Rightarrow C_V = \left(\frac{\partial E}{\partial T}\right) = 3N_A K \left(\frac{h\nu_0}{KT}\right)^2 \frac{e^{h\nu_0/KT}}{\left(e^{h\nu_0/KT} - 1\right)^2}$$

At low temp., $h\nu_0/KT \gg 1$

$$\Rightarrow \quad e^{h\nu_0/KT} \gg 1$$

Above eqn. reduces to

$$C_V = 3N_A K \left(\frac{h\nu_0}{KT}\right)^2 e^{-h\nu_0/KT}.$$

On decreasing the temp., the exponential factor decreases much more rapidly than the corresponding increase in the factor $(h\nu_0/KT)^2$.

$\Rightarrow$ C_V decreases with decrease in temperature.

At low temperatures, from Einstein eqn —

At high temp

$$\bar{E} = \frac{h\nu_0}{e^{h\nu_0/KT} - 1}$$

The value of C_v reduces to the classical value of $3R$.

As at high temp. $h\nu_0/KT \ll 1$.

Applying expansion series $e^x = 1 + x + x^2 + \cdots$ eqn 3

becomes $= 3N_A K \left(\frac{h\nu_0}{KT}\right)^2 \frac{1 + (h\nu_0/KT) + \cdots}{\left\{1 + (h\nu_0/KT) + \frac{1}{2}(h\nu_0/KT)^2 + \cdots\right\}^2}$

$$= 3N_A K \left(\frac{h\nu_0}{KT}\right)^2 \left[\frac{1 + (h\nu_0/KT) + \cdots}{\left\{(h\nu_0/KT) + \frac{1}{2}(h\nu_0/KT)^2 + \cdots\right\}}\right]$$

$$= 3N_A K \frac{(h\nu_0/KT)^2 + (h\nu_0/KT)^3 + \cdots}{\left[h\nu_0/KT + \frac{1}{2}(h\nu_0/KT)^2 + \frac{1}{3}(h\nu_0/KT)^3 + \cdots\right]^2}$$

At high temp, $\frac{h\nu_0}{KT}$ is v.v. small $\Rightarrow$

Powers ≥ 2 can be neglected.

$$C_v = 3N_A K = 3R \qquad \left(\frac{R}{N} = K\right)$$

$$\boxed{C_v = 3R}$$ — Classical value